Parameter Independence and Outcome Independence in Dynamical Collapse Theories

Travis Norsen

Marlboro College

Marlboro, VT 05344

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Abstract

Ghirardi, Grassi, Butterfield, and Fleming have previously argued that two distinct formulations of dynamical reduction (GRW) theories have distinct non-local properties: one (the "non-linear" formulation) violates Parameter Independence while the other (the "linear + cooking" formulation) violates Outcome Independence. The claim is then that, since theories violating Outcome Independence are compatible with relativity in a way that theories violating Parameter Independence are not, the "linear + cooking" dynamical reduction theories hold the key to reconciling quantum non-locality with relativity. I review and assess this claim, arguing that, while ultimately misleading, there is an interesting and important lesson to be extracted.

I. INTRODUCTION

In an earlier paper [1] I criticized Jon Jarrett's distinction [2] between (what Shimony [3] later dubbed) "Parameter Independence" (PI) and "Outcome Independence" (OI). Jarrett had shown that Bell's local causality condition [4] was equivalent to the conjunction of PI and OI, and he argued that PI *alone* captured relativity's alleged prohibition on superluminal causation. This suggested in turn that candidate quantum theories violating OI (such as orthodox QM and the GRW spontaneous collapse theory) should be preferred over theories violating PI (such as the de Broglie - Bohm pilot-wave theory) because of the former's favorable compatibility with relativity theory. It was brought out in Ref. [1], however, that Jarrett's argument was based on a simple confusion about Bell's local causality condition; correcting the confusion clarified that violations of OI and PI are equally troubling from the point of view of relativity, suggesting in turn that the very distinction between OI and PI has no useful role to play in guiding ongoing attempts to reconcile quantum theory and relativity.

In subsequent private communication, however, GianCarlo Ghirardi pointed out to me two papers [5, 6] he had coauthored with R. Grassi, J. Butterfield, and G. Fleming in which the authors (GGBF) show that two different mathematical formulations of the GRW-type dynamical collapse theory (in the context of non-relativistic quantum mechanics) have distinct non-local properties: one (the "non-linear" formulation) violates PI while the other (the "linear + cooking" formulation) violates OI.

The alleged implication was that, because – according to Jarrett and, by a rather different argument, also Ghirardi and Grassi [7] – theories violating OI are supposed to be more compatible with relativity than theories violating PI, one should preferentially pursue "linear + cooking" theories in the quest for a genuinely relativistic quantum theory without observers. My intuition was that this could not possibly be correct, for three reasons: first, Jarrett's argument for the relevant premise (that theories violating OI are more compatible with relativity than theories violating PI) is simply mistaken; second, the "non-linear" and "linear + cooking" formulations of GRW seem somehow obviously to be just two mathematically distinct ways of presenting the same theory, making it implausible that they should exhibit meaningfully different types of nonlocality; and third, since the publication by GGBF, a genuinely relativistic GRW-type theory has been produced [8] and its formulation is explicitly "non-linear" (as opposed to "linear + cooking"). (Though its author, Roderich Tumulka, assures me it could also be formulated in "linear + cooking" terms.)

Sorting all of this out, however, turned up some interesting points. The purpose of this note is simply to share them.

II. TWO FORMULATIONS OF GRW

The main motivation for the GRW-type dynamical collapse theories is to solve the measurement problem. Where orthodox QM contains two distinct rules for the time-evolution of state vectors (the Schrödinger equation and the collapse rule) and so involves an allegedly fundamental (but in fact vague and ill-defined) distinction between processes that are "measurements" and those that aren't, GRW incorporates a stochastic collapse process into the Schrödinger equation itself, thus allowing a single uniform dynamics for the wave function which applies all the time (regardless of whether a "measurement" is happening or not). See Ref. [9] for the original proposal, and Ref. [10] for a recent review. Ref. [11] contains further discussion, partially in response to the already-cited Ref. [1], of the prospects of reconciling quantum non-locality with relativity within a dynamical collapse picture.[1]

We will be specifically concerned with the application of this theory to the usual kind of EPR-Bell situation. Suppose in particular that a pair of spin-1/2 particles in the spin singlet state is produced by a source, with each particle subsequently flying toward spatially-separated detectors on the left (L) and right (R) where they are subjected (or perhaps not subjected) to measurements of some spin component. For our purposes it will suffice to consider the same restricted situation considered by GGBF: the experimenter on the left always measures his particle along a fixed direction, while the experimenter on the right chooses either to make a measurement along that same fixed direction (which we denote $\hat{a}$) or to make no measurement at all (which we denote $*$).

For this situation, Parameter Independence is the requirement that, for example, the probability

[1] Perhaps I can also take the opportunity here to clarify a misunderstanding that runs through Ref. [11], based on some sloppiness in my own earlier paper. In Ref. [1], my aim was principally to refute Jarrett's claim that violation of Bell's local causality condition didn't necessary entail the existence of any superluminal causation – i.e., to refute Jarrett's claim that Bell's local causality condition was somehow too strong as an attempt to capture relativity's (alleged) prohibition on superluminal causation. I thus largely took for granted what Jarrett himself had taken for granted: that relativity does indeed genuinely prohibit causal influences between spacelike separated events. In fact, though, the connection between superluminal causation and relativity is more subtle. They are not necessarily incompatible, and I agree entirely with Ghirardi that, for example, Tumulka's recent theory [8] provides a very promising concrete example of how they can indeed be reconciled within a GRW framework. To summarize, where Ghirardi not unreasonably reads Ref. [1] as claiming an "irremediable conflict" between the empirically correct predictions of quantum theory and relativity as such, I in fact intended a rather different claim: that there is an "irremediable conflict" between the empirically correct predictions of quantum theory and the idea of exclusively sub-luminal causation that many people – *reasonably, but perhaps in the end wrongly* – have taken to be a requirement of relativity.

assigned to the outcome "up" for the experiment on the left, when conditioned on the initial state of the particle pair (and any other relevant "hidden variables"), should be independent of whether the experimenter on the right chooses $\hat{a}$ or $*$. That is, PI is the requirement that

$$P_\lambda(\text{up}_L|\hat{a}) = P_\lambda(\text{up}_L|*). \tag{1}$$

By contrast, Outcome Independence (OI) is the requirement that, when the setting $s_R \in \{\hat{a}, *\}$ of the distant apparatus is specified, the probability for a specific outcome on the left doesn't depend on the outcome $O_R \in \{\text{up}_R, \text{down}_R\}$ on the right. For example:

$$P_\lambda(\text{up}_L|s_R, O_R) = P_\lambda(\text{up}_L|s_R). \tag{2}$$

Jarrett showed, correctly, that Bell's local causality condition (that is, the condition we know now with reasonable certainty, based on experiments, to be false) is logically equivalent to the conjunction of PI and OI. Hence, empirically adequate theories must violate either PI or OI (at least if one allows freely-operating experimenters, sometimes called the "no conspiracy" assumption). (Jarrett was wrong, however, to think that theories violating PI necessarily contain superluminal causation, while those violating OI don't. See Ref. [1])

It will also be important later to appreciate that, for deterministic theories, the outcome O_R is necessarily a function of λ and the apparatus settings, so its specification is necessarily redundant (once these have been given). Hence, deterministic theories cannot violate OI and so must (if they violate Bell's local causality condition, i.e., if they are empirically viable) violate PI.

In the following two subsections, I briefly sketch a highly simplified version of each of the (allegedly distinct) formulations of GRW. In the next section I then discuss how each relates to OI and PI. See Ref. [5] for more details on how the "toy model" versions sketched here relate to the real GRW theory. The basic idea is to consider explicitly only the degrees of freedom associated with the entangled particle pair, and to simply assume the occurrence of a collapse when a measurement is made. This perhaps looks suspiciously like ordinary QM (with its distinct dynamical evolution laws for measurement and non-measurement situations), but it should be clear in the present context how the talk of collapses-upon-measurements should be understood from the GRW perspective. Note also that, to make the relevant conceptual points as clear as possible, various probabilities have been rounded off: for example, events which might occur (with tiny but nonzero probability) according to the real GRW theories are here assigned probability zero. One should therefore understand the relevant equations as involving only approximate equalities.

A. The "non-linear" formulation

GRW is a stochastic theory. For the purpose of indicating how the theory accounts for the experiment sketched above, it will suffice to mock up the stochastic aspect with a pair of coin flips – one on the right and one on the left. (This simplifying picture is adapted from discussions with Daniel Bedingham based on Ref. [12].) There is then a certain (non-linear) algorithm through which the incoming state vector, together with the results of the appropriate coin flips, determines how the state vector evolves through the measurements. Although the algorithm in principle is rather complicated – it is nothing but the (non-linear, stochastic) evolution equation that replaces the Schrödinger equation for GRW – we can present here a much simplified algorithm that captures (to the accuracy described above and for the limited purpose of analyzing the simple EPR experiment at hand) all the relevant aspects of the theory.

Here is the algorithm. For a given measurement, a certain quantum state $|\psi_{in}\rangle$ will "enter" and interact with the measuring device. After the measurement, the device will register either "up" or "down" – and the quantum state will have collapsed to the corresponding eigenstate of the appropriate spin operator – as follows:

- If $\|\hat{P}_\uparrow|\psi_{in}\rangle\|^2 \gg \|\hat{P}_\downarrow|\psi_{in}\rangle\|^2$ then the device will register the result "up" and the quantum state will collapse to $|\psi_{out}\rangle = P_\uparrow|\psi_{in}\rangle/\|P_\uparrow|\psi_{in}\rangle\|$.

- If $\|\hat{P}_\uparrow|\psi_{in}\rangle\|^2 \ll \|\hat{P}_\downarrow|\psi_{in}\rangle\|^2$ then the device will register the result "down" and the quantum state will collapse to $|\psi_{out}\rangle = P_\downarrow|\psi_{in}\rangle/\|P_\downarrow|\psi_{in}\rangle\|$.

- If $\|\hat{P}_\uparrow|\psi_{in}\rangle\|^2 \approx \|\hat{P}_\downarrow|\psi_{in}\rangle\|^2$ then the device will register the result "up" or "down" (and the state will collapse in the associated way) according to whether the coin flip comes up "heads" or "tails".

Here, the projection operators $\hat{P}_\uparrow$ and $\hat{P}_\downarrow$ onto the spin-up and spin-down states (respectively) should be understood as acting on the degrees of freedom associated with whichever particle is being measured (L or R).

Notice that the coin flip is only used, so to speak, as a tie-breaker. If the incoming quantum state is already dominantly spin-up or spin-down, the evolution simply preserves this property (and the measuring device reveals it, and the coin flip is irrelevant). But if the incoming quantum state is a balanced superposition of spin-up and spin-down, then the coin flip determines the outcome and the quantum state collapses accordingly.

Notice also that the algorithm is both stochastic – the outcome of the measurement is (in the general case) influenced by the result of the coin flip – and non-linear: the algorithm itself "looks at" the incoming state vector in order to decide how (if at all) the state vector should evolve through the course of the measurement interaction.

Note finally that, in the event that no measurement occurs, there is by definition no interaction between the particle pair and the measuring device, in which case $|\psi_{out}\rangle$ will necessarily just equal $|\psi_{in}\rangle$.

Let's now see how this works in the EPR-Bell scenario at hand. We consider a Lorentz frame in which the experiment on the right happens first. Then the quantum state $|\psi^R_{in}\rangle$ which enters the device on the right is just the initial singlet state:

$$|\psi_0\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow L, \downarrow R\rangle - |\downarrow L, \uparrow R\rangle \right). \tag{3}$$

In the case that the experimenter on the right chooses to actually make a measurement, the algorithm sketched above will necessarily go to the tie-breaker – that is, the outcome ("up" or "down") will be determined by the result of the coin flip (H_R or T_R, respectively) and the quantum state will collapse accordingly. Thus, the quantum state which subsequently enters the device on the left (ignoring the overall pure phases which are irrelevant here) will be given by

$$|\psi^L_{in}\rangle = \begin{cases} |\downarrow L, \uparrow R\rangle & \text{if } H_R \\ |\uparrow L, \downarrow R\rangle & \text{if } T_R \end{cases} \tag{4}$$

It then follows from the above algorithm that the coin flip on the left will necessarily be irrelevant to the outcome there: the outcome on the left will be "up" or "down" according to whether the state vector is already (respectively) a spin-up or spin-down eigenstate, i.e., according to whether (respectively) T_L or H_L was realized on the right.

This scheme thus clearly reproduces the familiar quantum mechanical statistics for both the individual and joint outcomes of the two experiments.

Now we must also consider the possibility that the experimenter on the right chooses instead not to make a measurement at all. In that event, no collapse occurs prior to the measurement on the left and so:

$$|\psi^L_{in}\rangle = |\psi_0\rangle. \tag{5}$$

And this of course implies, again according to the above algorithm, that the coin flip on the left will now determine the outcome there, with the outcome being "up" if the result is H_L and "down" if the result is T_L. And so the scheme reproduces the correct statistics for this scenario, too.

Though both simplified and schematic (even relative to the already simplified and schematic version in GGBF) this model accurately captures all the relevant aspects of the "non-linear" GRW theory's explanation for the measurement statistics in this kind of experiment.

B. The "linear + cooking" formulation

Let us now turn to the alternative "linear + cooking" formulation of GRW. The idea here is to replace the explicitly non-linear (stochastic) dynamics of the previous formulation, with a fully linear dynamics that is then supplemented by some post-hoc readjustments of the probabilities. As before, we will explain the dynamics by tracing the evolution of the quantum state vector through its interaction with the two measuring devices, assuming a Lorentz frame in which the device on the right is encountered first. And as before, the stochastic element of the theory will be captured by a pair of coin flips, one on each side. The main difference with the "non-linear" formulation will be that here the coin flip will *always* determine the measurement outcome and the state to which the incoming state collapses (as opposed to being used only in the event of a tie).

Let's again see how this works by following the evolution through the two measurements, starting with the case that the experimenter on the right does decide to make a measurement. As indicated, the outcome of the experiment is determined by an un-biased coin flip, with H_R producing a result "up$_R$" and T_R producing a result "down$_R$". In addition, the state vector collapses, though (for reasons that will become clear shortly) we will here impose the collapse by acting with the appropriate projection operator – but *not* re-normalizing the state. Thus, the quantum state later entering the device on the left will be given by

$$|\psi_{in}^L\rangle = \begin{cases} -|\downarrow L, \uparrow R\rangle/\sqrt{2} & \text{if } \ H_R \\ |\uparrow L, \downarrow R\rangle/\sqrt{2} & \text{if } \ T_R \end{cases} . \tag{6}$$

The outcome on the left is then also determined by an unbiased coin flip, with H_L indicating "up$_L$" and T_L indicating "down$_L$". It is also necessary now to consider the further state vector collapse associated with the measurement on the left: the final quantum state (after interaction with both measuring devices, but without imposing a re-normalization after the collapses) is

$$|\psi_{final}\rangle = \frac{1}{\sqrt{2}} \begin{cases} \hat{P}_{\uparrow L}\hat{P}_{\uparrow R}|\psi_0\rangle = |\emptyset\rangle & \text{if } \ H_L\, H_R \\ \hat{P}_{\uparrow L}\hat{P}_{\downarrow R}|\psi_0\rangle = |\uparrow L, \downarrow R\rangle & \text{if } \ H_L\, T_R \\ \hat{P}_{\downarrow L}\hat{P}_{\uparrow R}|\psi_0\rangle = -|\downarrow L, \uparrow R\rangle & \text{if } \ T_L\, H_R \\ \hat{P}_{\downarrow L}\hat{P}_{\downarrow R}|\psi_0\rangle = |\emptyset\rangle & \text{if } \ T_L\, T_R \end{cases} \tag{7}$$

where $|\emptyset\rangle$ is the zero vector. The two coin flips were un-biased *and independent*, so each of the four possible joint outcomes ($\{H_L, H_R\}$, $\{H_L, T_R\}$, $\{T_L, H_R\}$, and $\{T_L, T_R\}$) have equal probabilities of 1/4, as (consequently) do each of the four possible joint outcomes of the experiments ("up on the left and up on the right", and so on).

These, however, represent only the "raw" probabilities. We now readjust these joint probabilities – i.e., "cook" – by re-weighting the probability of each joint outcome according to the squared modulus of the associated final quantum state. Two of these possible final states have zero norm, and the other two have equal norms, so the upshot of the "cooking" is the assertion that, at the end of the day,

$$\begin{aligned} P_{cook}(\mathrm{up}_L, \mathrm{up}_R) &= 0 \\ P_{cook}(\mathrm{up}_L, \mathrm{down}_R) &= 1/2 \\ P_{cook}(\mathrm{down}_L, \mathrm{up}_R) &= 1/2 \\ P_{cook}(\mathrm{down}_L, \mathrm{down}_R) &= 0 \end{aligned}$$

so that we again reproduce the usual quantum mechanical statistics for both the individual and joint outcomes.

Now let us finally walk through the "linear + cooking" formulation in the case that the experimenter on the right chooses not to make any measurement. The only difference is that (since there is no coupling between the experimental apparatus on the right and the particle pair) the state of the pair which enters the apparatus on the left is the original singlet state. There are then only two possible final state vectors – the same two which $|\psi_{in}^L\rangle$ might have been had the experimenter on the right made a measurement – and they have equal norms. So the "raw" and "cooked" probabilities turn out to be equal:

$$\begin{aligned} P_{cook}(\mathrm{up}_L) &= 1/2 \\ P_{cook}(\mathrm{down}_L) &= 1/2 \end{aligned} \tag{8}$$

and we thus reproduce the expected 50/50 chances for the two possible outcomes on the left.

III. EQUIVALENCE AND NON-LOCALITY

Not too much analysis is necessary to make it clear that the "non-linear" and "linear + cooking" formulations are completely equivalent. In the event that the experimenter on the right makes a

measurement, the predictions of the "non-linear" formulation come down to the following set of probability statements:

$$
\begin{aligned}
P(\mathrm{up}_R) &= 1/2 \\
P(\mathrm{down}_R) &= 1/2 \\
P(\mathrm{up}_L|\mathrm{down}_R) &= 1 \\
P(\mathrm{down}_L|\mathrm{up}_R) &= 1
\end{aligned}
$$

with $P(\mathrm{up}_L|\mathrm{up}_R) = P(\mathrm{down}_L|\mathrm{down}_R) = 0$.

On the other hand, the "linear + cooking" formulation comes down instead to the following:

$$
\begin{aligned}
P(\mathrm{up}_L, \mathrm{down}_R) &= 1/2 \\
P(\mathrm{down}_L, \mathrm{up}_R) &= 1/2
\end{aligned}
$$

with $P(\mathrm{up}_L, \mathrm{up}_R) = P(\mathrm{down}_L, \mathrm{down}_R) = 0$.

But these two sets of probability statements are completely equivalent. (And the corresponding sets for the case that the experimenter on the right *doesn't* make a measurement, are even more trivially equivalent.) So we are dealing here with two different-looking recipes which generate the same one stochastic dynamics for the same set of beables. There are not two distinct theories here, only two distinct presentations of the same theory.

It should also be clear that the "linear + cooking" theory isn't really linear at all – it's just that the non-linearity (which, here, means dependence of dynamical probabilities for what happens to the quantum state, on the quantum state itself) is introduced "at the end" (in the "cooking" process) as a post-hoc readjustment of the relative likelihoods for the various things that (otherwise) could have happened. It may be that one or the other of the two ways of mathematically formulating the theory has some advantage in the context of trying to produce relativistic generalizations from the non-relativistic theory. For example, Lorentz covariance may be somewhat more manifest (all other things being equal) in a "linear + cooking" formulation than in a "non-linear" formulation. (See Ref. [12].) But this should not be confused with the two theories being somehow physically different. Manifest or not, the two formulations would seem to have to have the same ultimate status *vis a vis* Lorentz invariance – and, it would seem, also (in the non-relativistic context) *vis a vis* nonlocality and the PI/OI distinction.

IV. GGBF'S NONLOCALITY ANALYSIS

The main claim of GGBF, however, is that this is not so: they allege that the "non-linear" formulation violates PI, while the "linear + cooking" formulation instead violates OI.

We begin with GGBF's straightforwardly correct demonstration that the "linear + cooking" formulation respects PI. The proof is just that the marginal probability, for (say) an "up" outcome on the left, is the same – namely 1/2 – regardless of whether the experimenter on the right makes a measurement or not. This is incontestably correct, and it implies that the "linear + cooking" formulation violates OI. (This can be seen directly as well: for the "linear + cooking" theory $P_{|\psi_0\rangle}(\mathrm{up}_L|s_R) = 1/2$ while $P_{|\psi_0\rangle}(\mathrm{up}_L|s_R, O_R)$ is either 0 or 1 or 1/2 depending on the realized values of s_R and O_R.)

Given all that's been said, then, the puzzle is that GGBF claim also to prove that the "non-linear" formulation violates PI instead. What is their argument?

The key to GGBF's argument for this claim is their taking of the coin flip on the left as a kind of "hidden variable" which, in conjunction with the incoming state vector, determines the probabilities for the various possible outcomes. To illustrate, one may consider for example the case in which this coin flip on the left happens to give "heads" (i.e., the case H_L). Then the λ in Equation (1) subsumes both $|\psi_0\rangle$ and H_L. If the experimenter on the right chooses to make a measurement, we then have that

$$P_\lambda(\mathrm{up}_L|\hat{a}) = P_{|\psi_0\rangle}(\mathrm{up}_L|\hat{a}, H_L) = 1/2 \tag{9}$$

because, while it's given that an experiment happened on the right so that the state vector which enters on the left will be collapsed, it is unknown which outcome this collapse produced, so we must average over the two (equiprobable) possibilities.

On the other hand, if the experimenter chooses *not* to make a measurement, we have that

$$P_\lambda(\mathrm{up}_L|*) = P_{|\psi_0\rangle}(\mathrm{up}_L|*, H_L) = 1 \tag{10}$$

since the non-performance of a measurement on the right ensures that the algorithm generating the outcome on the left will go to the tie-breaker, in which case the (given) H_L guarantees the outcome "up_L".

Since $1/2 \neq 1$ we have a violation of PI. That, at least, is the claim made by GGBF, based on essentially the argument just sketched.

But one thing should be immediately clear: this argument hinges crucially on taking the outcome of the coin flip on the left as a hidden variable, i.e., as something that gets brought into the

calculation of probabilities through λ. (If this is not done, the "non-linear" formulation, like the "linear + cooking" one, violates OI instead.) And, arguably, this is a completely wrong thing to do, because doing it effectively converts GRW (which is supposed to be an irreducibly stochastic theory) into a deterministic hidden variable theory. And we know, from the beginning, that deterministic theories (which violate Bell's local causality condition) must necessarily violate PI rather than OI.

So in that sense the puzzling result is no surprise. It is just a consequence of an inappropriate way of thinking about stochastic theories. How should we think about them instead? We should regard the "coin flips" (more generally, the random numbers that play some role in the dynamics) not as *beables* (to use Bell's terminology for things that are physically real), but only as internal aspects of the algorithm which defines the dynamics *for* the beables.

V. DISCUSSION

Let us explore in a bit more detail the consequences of treating the coin flips as beables in the two formulations of GRW. To begin with, it should be clear that if one regards the coin flips the way I think they should be regarded – not as beables but as internal aspects of the algorithm which defines the dynamics *for* the beables – then both the "linear + cooking" and "non-linear" formulations of GRW respect PI and violate OI. Indeed, understood this way, it is clear that the two formulations are merely two different mathematical formulations of the same one physical theory: they posit the same beables and mathematically equivalent laws governing the beables.

As shown just above, however, if one takes the coin flips themselves as beables for the "non-linear" formulation, this becomes in effect a (non-local) deterministic hidden variable theory. That it violates PI rather than OI is then not terribly surprising or interesting.

One perhaps expects the same thing to occur with the "linear + cooking" formulation, but here there is something that is surprising and interesting. Recall that, in the "linear + cooking" formulation, the coin flip outcomes simply determine the corresponding measurement outcomes: $H_L \to \mathrm{up}_L$, and so on. So if we consider the outcomes of the coin flips as given hidden variables, we have for example that, for $\lambda = (|\psi_0\rangle, H_L)$,

$$P_\lambda(\mathrm{up}_L | s_R) = 1 \tag{11}$$

and that

$$P_\lambda(\mathrm{up}_L | s_R, O_R) = 1 \tag{12}$$

so that OI is respected. But

$$P_\lambda(\text{up}_L|\hat{a}) = P_\lambda(\text{up}_L|*) = 1 \tag{13}$$

so PI is respected as well! What is going on here?

The point is that, in the "linear + cooking" formulation, the non-locality is in the part of the dynamics that brings about some particular set of coin flip outcomes. (Recall that the coin flip outcomes are, after cooking, *correlated* despite their association with spacelike separated spacetime regions.) So if one regards these coin flip outcomes as simply given, the remaining dynamics (which in effect just has the experimental outcomes being read off from the coin flips) is completely local. It makes the theory into a kind of conspiracy theory, in that the otherwise-puzzling correlations between measurement outcomes are written in advance into some given (and so unexplained) correlations in the hidden variables. But still, the theory is local.[2]

To summarize the possibilities: by fiddling around with (a) which elements of a theory are granted "beable status" [4, p. 53] and (b) whether certain beables are regarded as simply given, or instead as unfolding in some dynamical process, GRW can be variously understood as a non-local stochastic theory (which violates OI but respects PI), a non-local deterministic theory (which respects OI and violates PI), or a local conspiracy theory (which respects both OI and PI).

From the point of view of Bell's careful formulation of local causality for candidate theories, none of this is terribly surprising. Indeed, Bell's locality criterion is "in terms of local beables" [4, p. 53] so it stands to reason that theories with different beables might have different standings with respect to local causality.[3]

Note also the connection between the mistake in GGBF's argument, analyzed above, and the ongoing controversy about the validity and implications of Conway and Kochen's so-called Free

[2] And note that, in this way of thinking about the theory, the conspiracy goes very deep. The probabilities for various coin flip outcomes depend (through the cooking) on the way the wave function evolves. And that in turn depends on the realized values of external fields, e.g., whether $\hat{a}$ or $*$ obtains. Thus, the notion of given, previously-assigned values for all the coin flip outcomes precludes the possibility of free-will choices or, equivalently here, free variables (such as "external fields"). This is not, therefore, a way of thinking about GRW that I think one should take seriously. It is in the category of theories Bell described as "superdeterministic".

[3] I made a point earlier of insisting that the "non-linear" and "linear + cooking" formulations were not different theories, on the grounds that they posited the same beables (namely just the wave function) and also mathematically equivalent laws for the dynamical evolution of those beables. The point here is that, by granting "beable status" to some additional elements in the theory (such as our coin flips), one *does* produce a genuinely new and distinct theory, and the question of *its* status vis a vis locality, OI, PI, etc., must be addressed anew. As a parallel example, consider orthodox one-particle quantum mechanics. This is a non-local theory, as shown most easily by the "Einstein's Boxes" argument. [13] But by adding an additional beable – the actual position of a particle within the wave – one has a new theory which, at least in the Einstein's Boxes scenario, is perfectly local. All of this illustrates the importance of recognizing that Bell's local causality condition applies to *candidate theories* and is "in terms of local beables".

Will Theorem. Conway and Kochen argue, wrongly, that a stochastic theory will necessarily have the same status (*vis a vis* local causality and/or relativity) as the deterministic theory it would be converted into by taking the background random numbers as beables. As they put this claim, we can

> "let the stochastic element ... be a sequence of random numbers (not all of which need be used by both particles). Although these might only be generated as needed, it will plainly make no difference to let them be given in advance." [14]

That in fact it *does* make a difference has been pointed out lucidly in Ref. [15]. (The authors show that, in their terminology, Tumulka's rGRWf theory is "effectively causal" – whereas the deterministic theory it would be converted into by taking all the random numbers as "given in advance" *isn't*.)

It is hoped that the analysis of the (simpler) example of this same error – discussed in the current paper – will help clarify and underscore this important lesson.

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